Extracting fundamental periods to segment human motion time series*

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The paper addresses a problem of sensor-based time series segmentation as a part of human activity recognition problem. We assume that each studied time series contains a fundamental periodic which can be seen as an ultimate entity (cycle) of motion. Due to the nature of the data and the urge to obtain interpretable results of segmentation, we define the segmentation as a partition of the time series into the periods of this fundamental periodic. To split the time series into periods we select a pair of principal components of the Hankel matrix. We then cut the trajectory of the selected principal components by its symmetry axis, thus obtaining half-periods that are merged into segments. A method of selecting a pair of components, corresponding to the fundamental periodic is proposed.

Keywords: *human activity recognition, time series segmentation, period detection, principal components analysis.*

Introduction

Recent advances in wireless technologies make it possible to obtain significant amounts of human-driven data describing everyday human activity with the help of various devices, such as mobile phone sensors. The analysis of human motion data allows to solve problems involved in health applications [10, 15]; in analysis of human behavior and social interactions [11, 12].

In this paper we consider time series of human motion [13]. We focus on the data which consists of measurements of acceleration in x, y and z dimensions as the person moves. We suggest the following plan of human motion data analysis:

1. *Segmentation.* Segmenting human motion time series is an important step in understanding an modelling human motion. From this point of view, the segmentation procedure has to produce interpretable results in terms of the nature of the segmented time series. We suggest that an interpretable segment can be seen as an entity of motion.

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During the first step, we divide the time series into relatively short segments. The key importance of this step is that the segmentation results are interpretable (in the sense of the nature of the process). By this we mean that each segment should correspond to a fundamental period of the studied TS to be seen as a basic unit of human motion. This aim is achieved by choosing several principal components as the features of each data point. The segmentation problem is then stated as a clustering problem.

2. *Clustering.* The clusterization problem is stated for the set of extracted periods. Each cluster should also be interpretable as a certain type of human motion (walking, jogging and so on).
3. *Designing a forecasting model.* For each cluster a forecasting model is designed. Each cluster is described by its own model.
4. *Online forecasting.* By default, each new sample that comes to the sample set, is treated as an object of the most recent cluster detected. The forecast is thus obtained with the model of this most recent cluster.
5. *Detecting the discord of a model.* Analysing the deviation of the real data from the forecasted value, the discord of the model is detected. The discord takes place when motion changes and the current model can no longer forecast the time series effectively.

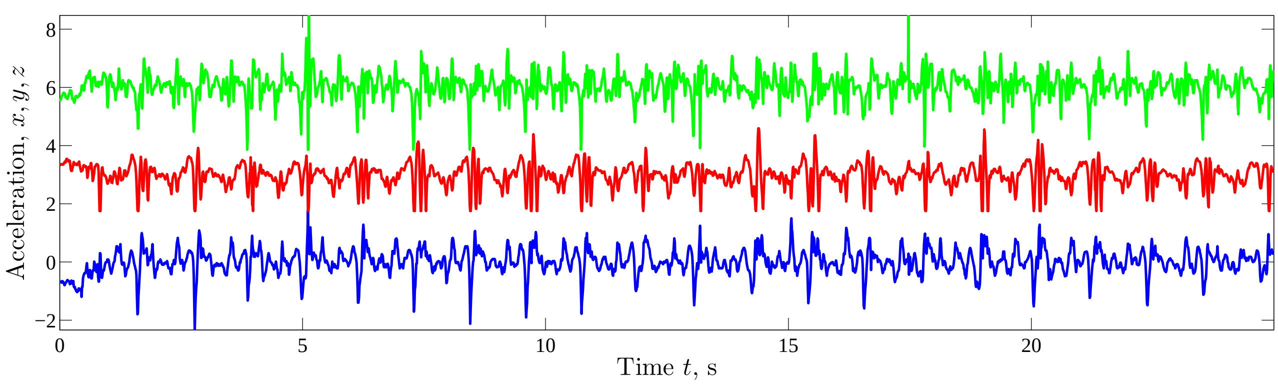

Fig. 1. An example of the human motion data: acceleration measurements for walking.

In this paper we focus on the first of the listed steps. The complexity of the segmentation problem lies in variety of studied time series evoked with the dependance on the person's physical abilities, irregularities in periodices and time scales of human motion. We state the problem of time series segmentation as a clusterization of time points problem.

The initial methods of evaluating frequency of a time series signal were based on approximating the time series with a sine model using least square estimation [1] or an asymptotically equivalent method — maximization of the periodigram [2]. Though the estimations obtained through these methods converge to the true value of frequency for strictly periodical time series, these methods are generally not applicable for quasi-periodic time series. Methods of estimating instantaneous frequency of a quasi-periodic signal are listed, for example, in [4] and include phase differencing, least square phase estimation, discrete-time Hilbert transform and others [4]. For example, the authors of [3] estimate the fundamental (i.e, the lowest frequency) as a fixed point of a mapping based on STFT, minimizing the self-introduncted Carrier-to-noise ratio. The paper also [4] explains how the earlier introduced methods for processing continuous signals can be used for evaluating the instantaneous frequency of a discrete time series. When dealing with quasi-periodic time series that describe human motion, one might enhance the results of period extraction using the known physical properties of the measured signal to directly detect steps on the time series. For example, the methods that aim to detect walking steps on accelerometer-based time series usually are designed to detect peaks in acceleration associated with heel strike or other phase of walking gate [5, 6]. The algorithms tested in the papers [5, 6] are derived from threshold methods when the the step is count every time the signal exceeds a predefined threshold [7], adaptive threshold methods [7], looking for a given pattern in a signal, window processing, transforming the signal into frequency domain [8, 14] and clustering the time- and frequency-domain features [9].

In this paper the segmentation problem is closely connected to the interpretability of the segments. The authors define their purpose as a partition of human motion time series into the segments that would be interpretable in a sense of mechanics of motion. Under the assumption that human motion time series are quasi periodic, the segmentation problem is regarded as the problem of period extraction. To solve this problem, we construct the trajectory matrix of the studied time series, and compute its principal components. We describe a procedure of selecting a pair of principal components corresponding to the fundamental periodic. The ending points of the periods are defined through cutting the trajectory of the chosen pair with its symmetry axis.

Problem statement

We consider periodical or quasi-periodical types of human motion (such as walking, running, leaping and so on) and suppose that each studied time series of human motion comprises a fundamental periodic, connected with the cycle of motion. This fundamental period is defined by assumptions on the nature of the motion. For example, during the cycle of normal human gait, each lower limb goes through a stance phase and a swing phase and then returns to the stance phase [16]. The sequence of times series points, measured with accelerometer during one gait cycle is repeated as the human walks on. The time scale of the cycle and the exact values of the acceleration may change, but the characteristic form of the signal stays the same (for example, every such segment of time series describing vertical component of acceleration contains two maximums: the first one corresponding to the landing of the heel and another one to pushing off the ground). We'll assume that the length of the period changes are small enough for us to consider the time series periodic. For our purposes it is unimportant where to start measuring the cycle of motion: when the heel was landing, or when it was taking off. It is only necessary that the cycle of walking starts and end with the same pose. Similarly to the walking gait we define the *fundamental period* of motion as a segment of the time series, measured between two consequential moments of time when a person's body takes the same pose. Several examples of human motion time series with fundamental periods plotted on them are given in the picture 6.

In this paper we solve the problem of segmentation of the time series $X = \{x(i)\}_{i=1}^m$, regarding a result of segmentation as a partition $\mathcal{I}_\mathcal{S} = \{i_1, \dots, i_T\}$ of the time series X into a sequence $\mathcal{S} = \{S_1, \dots, S_T\}$ of segments $S_t = [x(i_t), \dots, x(i_{t+1} - 1)]^\mathsf{T}$, each corresponding to a fundamental period. The method we propose can be used for partitioning any periodical time series into periods, but the term *fundamental period* carries physical sense explained above and is only used in application to human motion time series. Throughout the paper we use the term *segmentation* for any procedure of partitioning time series; the proposed method is referred to as *period extraction*. Similarly, the results of the segmentation in general are called *segments*; the results of period extraction are called *extracted periods*.

The difficulty in the formal definition of quality of the results of the period extraction consists in defining the distance function d between the extracted periods, for one because the extracted periods might have different size.

As the etalon period can not always be defined, for the labeled data (with known quantity T^* of steps/periods), we use the absolute deviation

$$|\bar{S}| - \frac{m}{T^*}.$$

Here $|\bar{S}|$ denotes the average size of the extracted periods

$$|\bar{S}| = \frac{1}{T}\sum_{t=1}^{T}|S_t|, \quad |S_t| = i_{t+1} - i_t.$$

Period extraction using principal components

The physical nature of the data allows to suppose the presence of a fundamental period. Extracting segments which correspond to fundamental periods we automatically guarantee the interpretability of the segmentation. In this section we describe the suggested method of partitioning the time series into periods.

Let the time series X be a decomposition

$$X = \hat{X} + \tilde{X} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\hat{X}$ — is the so-called trend, $\tilde{X}$ — is the periodic (or a sum of them), and $\boldsymbol{\varepsilon}$ denotes the noise. Each constituent of the decomposition (1) can be approximated with a high level of accuracy [17] with a combination of the principal components of the trajectory matrix $\mathbf{H}$ of the centered time series X

$$x(i) \mapsto x(i) - \frac{1}{m}\sum_{i=1}^{m} x_i$$

.

$$\mathbf{H} = \begin{pmatrix} x(1) & x(2) & \dots & x(N) \\ x(2) & x(3) & \dots & x(N+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(m-N+1) & x(m-N+2) & \dots & x(m) \end{pmatrix}$$

To achieve this one should compute the singular value decomposition of covariance matrix of $\mathbf{H}$

$$\frac{1}{N}\mathbf{H}^{\mathsf{T}}\mathbf{H} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{\mathsf{T}}, \quad \Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_N) \tag{2}$$

and find the principal components $\mathbf{y}_j$, corresponding to positive eigenvalues of $\mathbf{H}^{\mathsf{T}}\mathbf{H}$ according to

$$\mathbf{y}_j = \mathbf{H}\mathbf{v}_j.$$

We will explain the procedure of selecting the principal components necessary to extract the periodic later, in the Section “Automatic selection of the fundamental pair of principal components”.

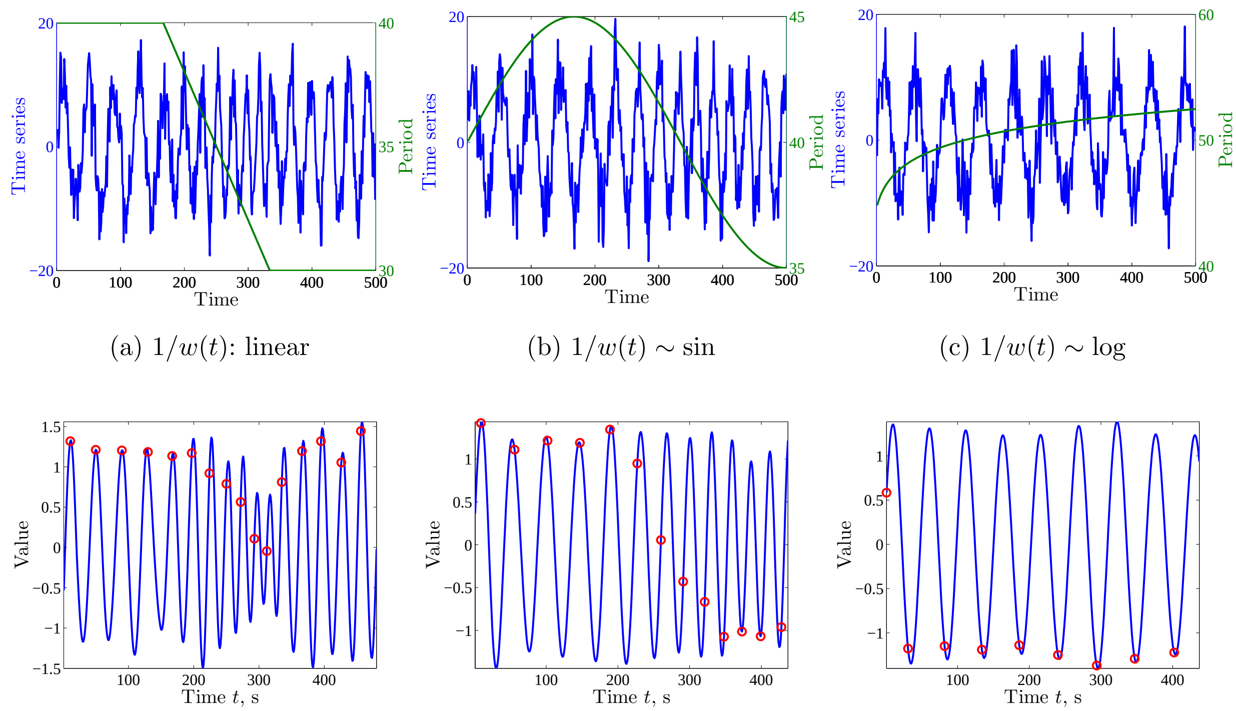

Fig. 2. Examples of segmenting phase-modulated time series.

Principal component selection

Each periodic constituent has a corresponding pair $(\mathbf{y}_j, \mathbf{y}_{j+1})$ of principal components (or, in more rare case, just one component) with approximately equal eigenvalues $\lambda_j \approx \lambda_{j+1}$. A pair of principal components defining a periodic can be determined by visual analysis: the trajectory of a pair of such pair should be approximately representable as a spiral (see, for example, figure 4) with fixed center and slightly varying radius

$$y_1(t) = r(\alpha t + \varphi)\cos(wt), \quad y_2(t) = r(\alpha t + \varphi)\sin(wt). \tag{3}$$

The paper [19] demonstrates that eigenvalues of principal components, corresponding to the same periodic constituent are asymptotically equal. The authors of [19] show this regarding the singular spectrum analysis in continuous notation. We will present the core of the proof using the denotation adopted in this paper. Let us consider for simplicity the case when the time series X consists from the only one periodic $\tilde{X}(t) = A\cos wt$. Then the (j,k)-th element of the matrix $\mathbf{H}^\intercal\mathbf{H}$ can be written as

$$\sum_{i=1}^{m-N+1} \cos\bigl(w(i+j-1)\bigr)\cos\bigl(w(i+k-1)\bigr) \tag{4}$$

According to the equation (2) the (j,k)-th element of the matrix $\mathbf{H}^\intercal\mathbf{H}$ can also be presented as

$$N\mathbf{v}_j^\intercal\mathbf{\Lambda}\mathbf{v}_k = N\sum_{i=1}^{N}\lambda_i v_{ji}v_{ki}. \tag{5}$$

Comparing the expression (4) to (5), we derive:

$$\sqrt{N\lambda_i}v_j(i) = \cos\bigl(w(i+j-1)\bigr) = \cos\bigl(w(i-1)\bigr)\cos(wj) - \sin\bigl(w(i-1)\bigr)\sin(wj)$$

Thus we have two principal components, namely a sine $A_i\sin wj$ and cosine $A_i\sin wj$ functions to describe this periodic. Then, considering normality $\mathbf{v}_j^\intercal\mathbf{v}_j = 1$ of the eigenvectors $\mathbf{v}_j$ we obtain for the eigenvalue λ_i

$$\lambda_i = \frac{A_i^2}{2N}\sum_{j=1}^{N}\cos^2 wj = \frac{A_i^2}{2}\sum_{j=1}^{N}(1-2\cos(2wj)).$$

The value of λ_j can be estimated as

$$\lambda_{\cos} \approx \frac{A^2}{2N}\int_0^N \bigl(1-2\cos(2wt)\bigr)dt = \frac{A^2}{2}\left(1-\frac{\sin(2wN)}{wN}\right) \quad \text{for } y_j \sim \cos wj,$$

$$\lambda_{\sin} \approx \frac{A^2}{2N}\int_0^N \bigl(1+2\cos(2wt)\bigr)dt = \frac{A^2}{2}\left(1+\frac{\sin(2wN)}{wN}\right) \quad \text{for } y_j \sim \sin wj.$$

When N is multiple of T, the periodic is described by the only principal component with the eigenvalue $\lambda = A^2/2$. If T does not divide N then, as N approaches infinity, the two eigenvalues $\lambda_{\cos}$ and $\lambda_{\sin}$ tend to the same value $\lambda_{\cos} = \lambda_{\sin} = A^2/2$.

This result shows that each periodic constituent can be approximated with two (when N is not multiple of T) principal components: the sine and cosine functions with the same frequency as the attached periodic and and asymptotically equal eigenvalues. This fact allows to consider only consequential pair in the procedure of automatical principal components selection, described in the Section "Automatical selection of a pair of principal components".

Dissection of the phase trajectory with its symmetry axis

Extracting the periods of a quasi-periodical constituent $\tilde{X}$

$$\tilde{X}(i) = f\left(\sum_{l=1}^{i} w(l) + \varphi_0\right)$$

of the time series X, one would like the ending points i_t to correspond to the same phase

$$\varphi(i_t) = \left(\sum_{i=1}^{i_t} w(i) + \varphi_0\right) \operatorname{mod} \frac{1}{w(i_t)} = \text{const}, \quad t = 1, \dots, T.$$

Suppose a pair $(\mathbf{y}_1, \mathbf{y}_2)$ of principal components of the time series X is fixed (or now let us omit the question of selecting a pair of components and assume the components are chosen optimally). If the instantaneous frequency $w(i)$ changes slowly over the size of the time series, the trajectory of the components constitutes a two-dimensional spiral with fixed center and slightly varying radius such that the closest points from different loops correspond to the same phase. Then to find the ending points of the periods we plot the trajectory of a pair of principal components (i.e. plot one component versus other) and cut it with a line that crosses the coordinate center ($y_2 = ky_1$ or $y_1 = 0$). Cutting the trajectory along this axis we obtain the partition $\mathcal{I}_{\mathcal{S}} = \{i_1^-, i_1^+, \dots, i_T^-, i_T^+\}$ of the time series into negative $S_t^- = [x(i_t^-), \dots, x(i_{t+1}^- - 1)]^\mathsf{T}$:

$$y_2(i) - ky_1(i) < 0 \text{ for all } i \in \{i_t^-, \dots, i_{t+1}^- - 1\}$$

$$(y_1(i) < 0 \text{ for all } i \in \{i_t^-, \dots, i_{t+1}^- - 1\})$$

and positive half-periods $S_t^+ = [x(i_t^+), \dots, x(i_{t+1}^+ - 1)]^\mathsf{T}$:

$$y_2(i) - ky_1(i) > 0 \text{ for all } i \in \{i_t^+, \dots, i_{t+1}^+ - 1\}$$

$$(y_1(i) > 0 \text{ for all } i \in \{i_t^+, \dots, i_{t+1}^+ - 1\}).$$

Assume $i_t^- < i_t^+$, than $i_t^- = i_t^+ - 1$. Joining half-periods S_t^- and S_t^+, we obtain the segmentation $\mathcal{I}_{\mathcal{S}} = \{i_1^-, \dots, i_T^-\}$ into fundamental periods. Thus we obtain half-periods that will be later merged into periods. Under the favorable conditions discussed above this procedure will provide us with the ending points of the same phase. In reality, $w(i)$ may change rather rapidly for human motion time series resulting in continuous bias of the extracted ending points i_t. The Figure demonstrates how different phase modulations affect the results of period extraction.

Another difference of processing the real data from the ideal case is that in reality the quality of period extraction depends highly on the slope of the cutting line. To reduce this effect we introduce the following heuristic: let us split the trajectory of the principal components with the line that is the closest to the symmetry axis of the studied trajectory. To find the symmetry axis we exploit the method of symmetrisation of a set of points proposed in [18]. Let the symmetry axis coincides with the ordinates axis $y_1 = 0$. According to [18], symmetrize a set of points

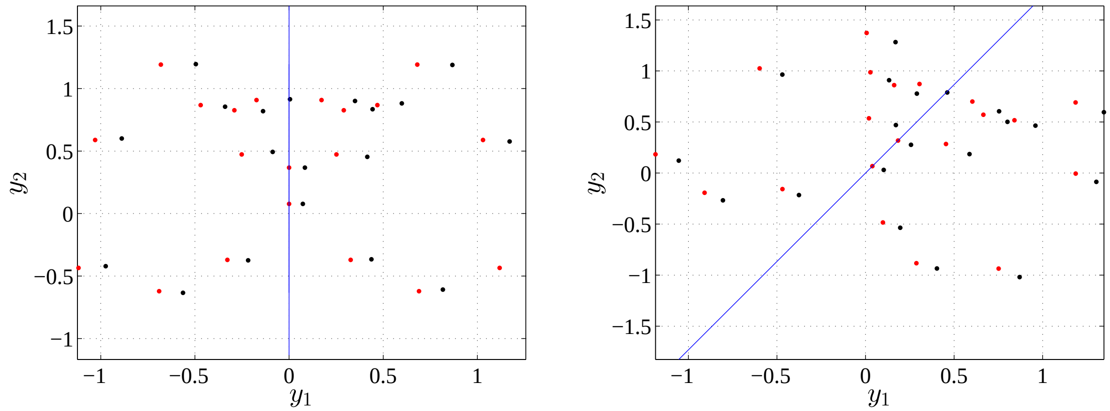

Fig. 3. Symmetrization of a set of points with respect to an axis covering the coordinate center.

$(\mathbf{y}_1, \mathbf{y}_2)$ an auxiliary vector $Y = [\mathbf{y}_1^\mathsf{T}, \mathbf{y}_2^\mathsf{T}]^\mathsf{T}$ is formed. Let the elements of Y be ordered so that for the symmetrized vector Y_s holds the following:

$$y_1(i) = -y_1(m' + i),\ i = 1, \dots, m',$$

$$y_2(i) = y_2(m' + i),\ i = 1, \dots, m',$$

$$y_1(i) = 0,\ i = 2m' + 1, \dots, m.$$

Then the symmetrization, minimizing the deviation

$$||Y_s - Y||_2$$

of symmetrised vector Y_s from the original vector Y is obtained through the transformation:

$$Y_s = \mathbf{Q}Y,$$

where

$$Q = \frac{1}{2}\mathbf{I}_{2n} + \frac{1}{2}\begin{pmatrix} -S & 0 \\ 0 & \mathbf{S} \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 & \mathbf{I}_m & 0 \\ \mathbf{I}_m & 0 & 0 \\ 0 & 0 & \mathbf{I}_{n-2m} \end{pmatrix}.$$

In general the symmetry axis is given by $y = \mathrm{tg}(\varphi)x$ or $x = 0$. We'll define the angle φ as the solution of a minimization problem

$$\varphi = \underset{\varphi \in [0, \pi/2]}{\arg\min} ||(Y_\varphi)_s - Y_\varphi||,$$

where Y_φ is the vector of coordinates of Y in the rotated by φ coordinate system. The picture 3 (on the left) presents a set of points and the results of its symmetrization with respect to the ordinates axis. The same set of points was rotated around the coordinates centre and then symmetrized. The results of symmetrization of the rotated set and the evaluated symmetry axis are presented by picture 3, on the right.

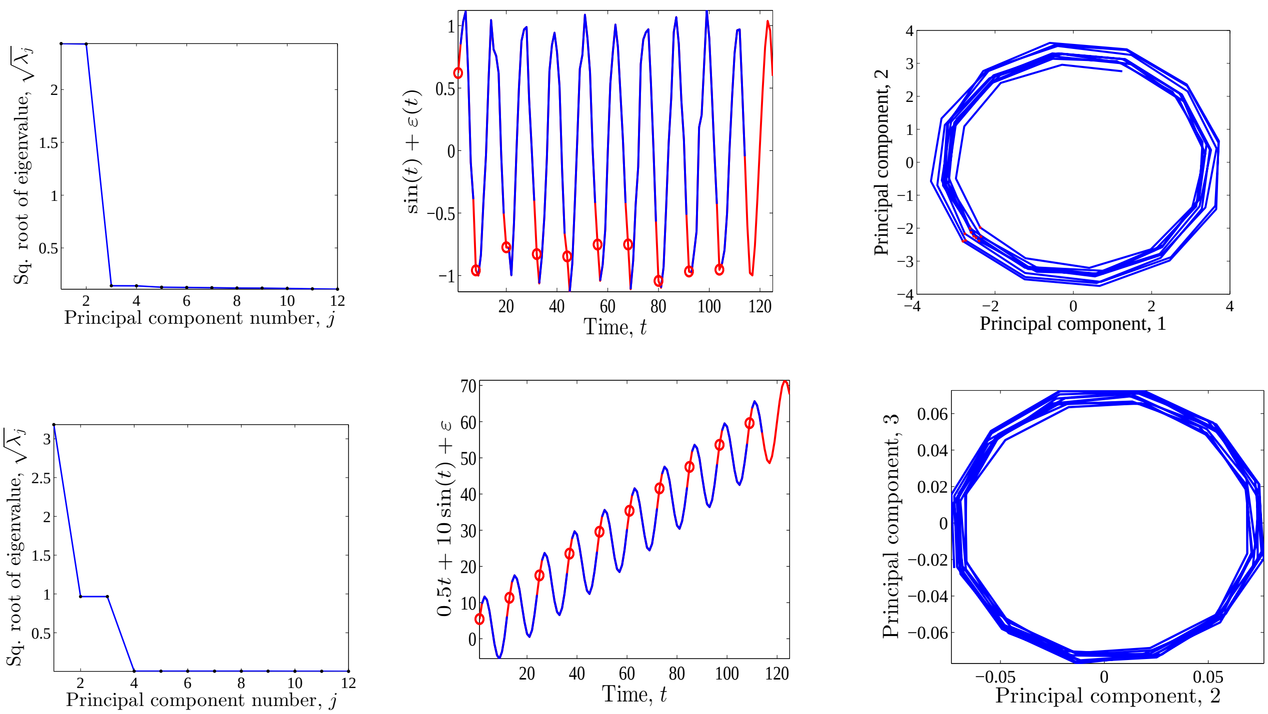

Fig. 4. The results of period extraction for the synthetic time series $X(t) = \sin(t) + \varepsilon$ and $Z(t) = = 0.5 + 10\sin(t) + \varepsilon$.

Example 1: period extraction on synthetical data. To demonstrate the proposed method of period extraction, let us consider two synthetical time series: $X(t) = \sin(t) + \varepsilon$ and $Z(t) = 0.5t + 10\sin(t) + \varepsilon$. To extract the periods we depict the time series X and Z with a pair of its principal components $\mathbf{y}_j(X)$ and $\mathbf{y}_j(Z)$ and cut the trajectory with symmetry axis to determine the starting and points of half periods and then merge them into periods. As it has already been mentioned, the simplest way to select the pair of principal components is visual analysis: for a pair with approximately equal eigenvalues we plot the trajectory of principal components. If the trajectory is similar to (3), then we select it as a periodic pair. The results of period extraction of the time series $X(t) = \sin(t)+\varepsilon$ and $Z(t) = 0.5t + 10\sin(t) + \varepsilon$ based on the pair of principal components selected this way are presented on the Figure 4. The pictures in the first column of 4 demonstrate how the values $\sqrt{\lambda_j}$ decrease. The time series $X(t)$ contains

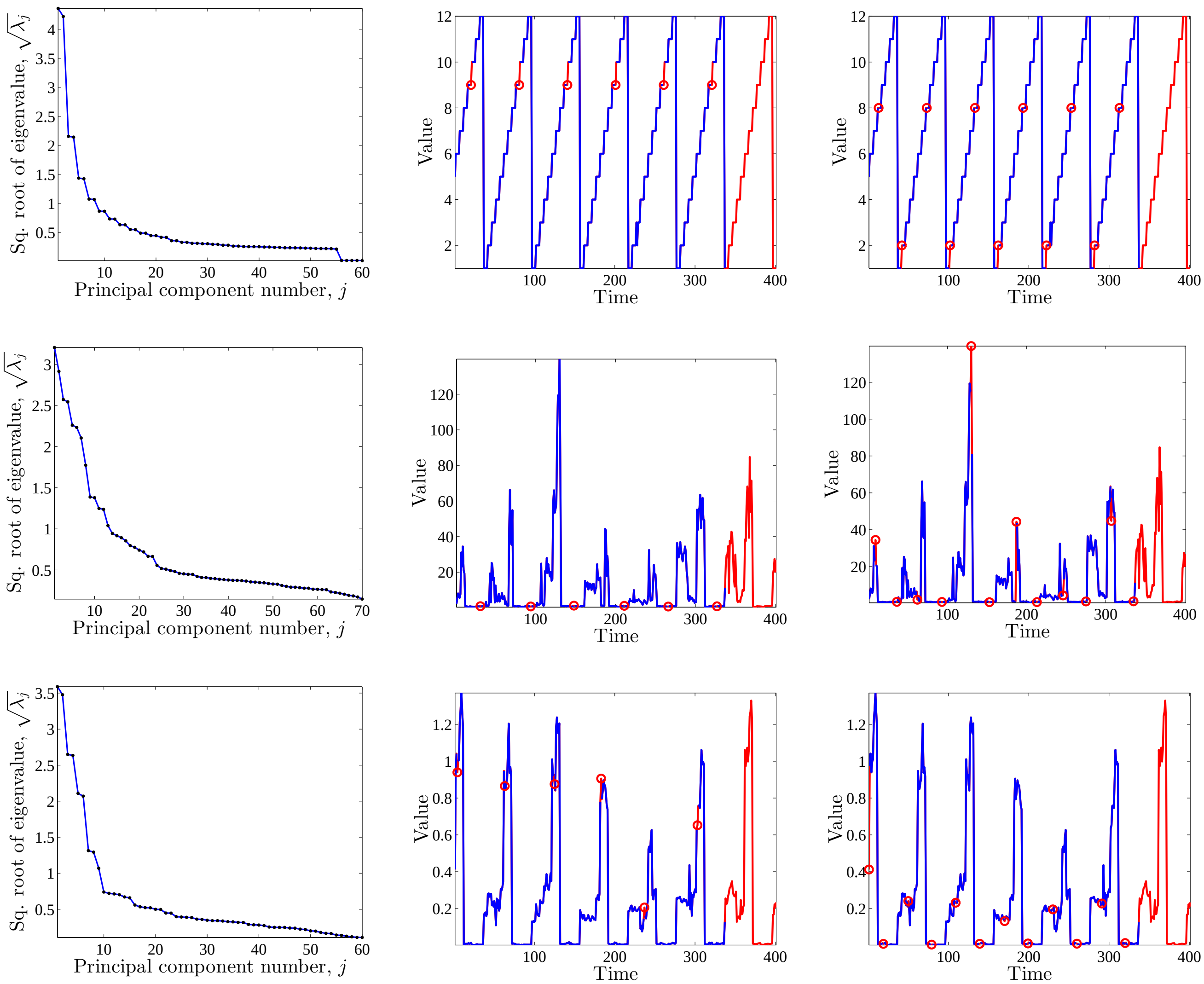

Fig. 5. Visual analysis of principal components: the eigenvalues of trajectory matrix and the results of period extraction based on the first and the second periodic pair ($\mathbf{y}_1$, $\mathbf{y}_2$) and ($\mathbf{y}_3$, $\mathbf{y}_4$) of principal components.

only one periodical constituent, thus the first two components were selected. The time series $Z(t) = 0.5t + 10\sin(t) + \varepsilon(t)$ consists of the periodic $\tilde{Z}(t) = 10\sin(t)$ and of the trend $\hat{Z}(t) = = 0.5t$. The pictures show that the first principal component $\mathbf{y}_1$ correspond to the trend $\hat{Z}$, while the second and the third are corresponding to the periodic $\tilde{Z}$.

Example 2: principal component selection for the fundamental period extraction on the data from [13] Although the visual analysis works well for this simple example, this method, but from other evident faults, has a limitation connected with the need to chose

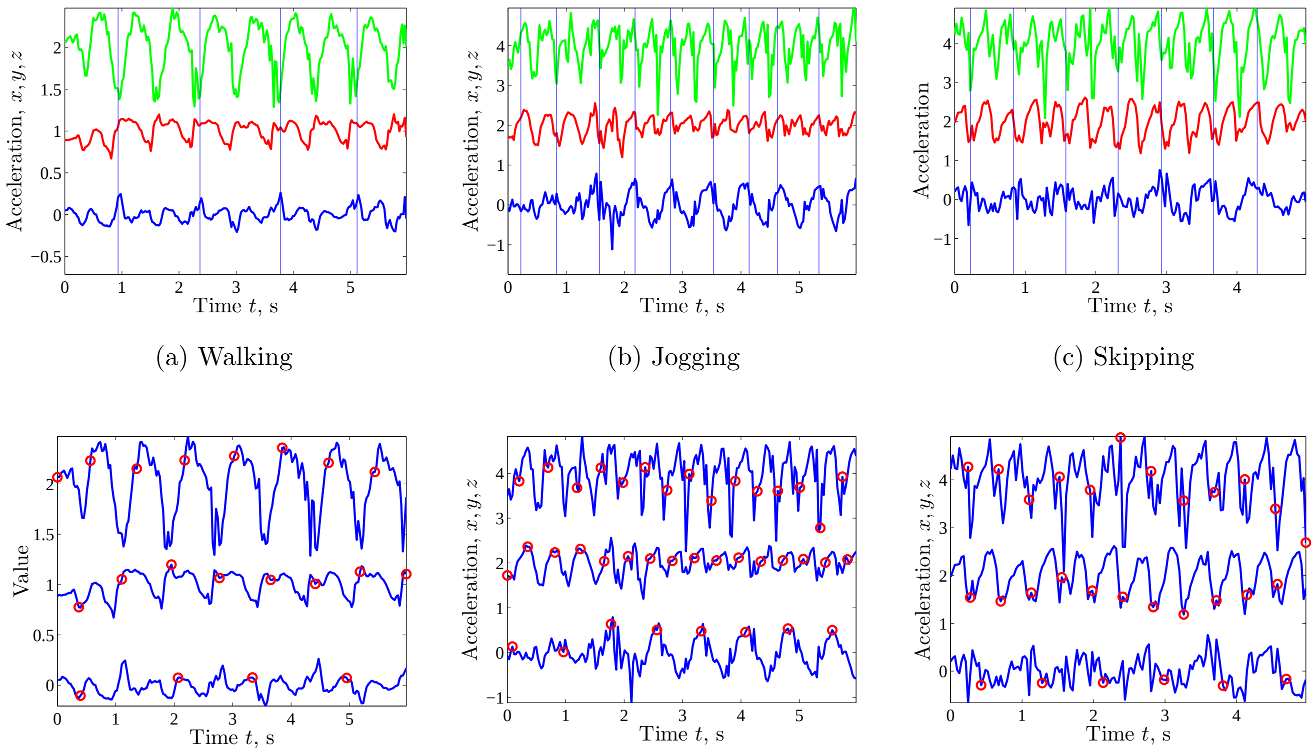

Fig. 6. (Top) Manually segmented time series: vertical lines divide the time range into periods. (Bottom) Segmentation results: red points mark the limits of the extracted segments.

between several periodic components. One possible solution is to chose the pair with maximum eigenvalues, assuming that principal components with lower eigenvalues approximate periodics with higher frequencies. This approach was implemented to the data [13], consisting of several periodical constituents. The results of processing and segmenting the time series are pictured on the Figure 5. The first column contains the square roots $\sqrt{\lambda_j}$ of the eigenvalues. It can be seen that each time series has several pair-candidates. The last columns of the Figure demonstrate the results of segmentation of the time series based on the first $(\mathbf{y}_1, \mathbf{y}_2)$ and the second $(\mathbf{y}_3, \mathbf{y}_4)$ periodical pair for each time series. The blue line is the historical time series, the red circles mark the starting/ending points of the segments. The red line correspond to a part of the time series that did not fit into the trajectory matrix $\mathbf{H}$. As the picture imply, the pairs with lower values of eigenvalues lead to more frequent segmentation.

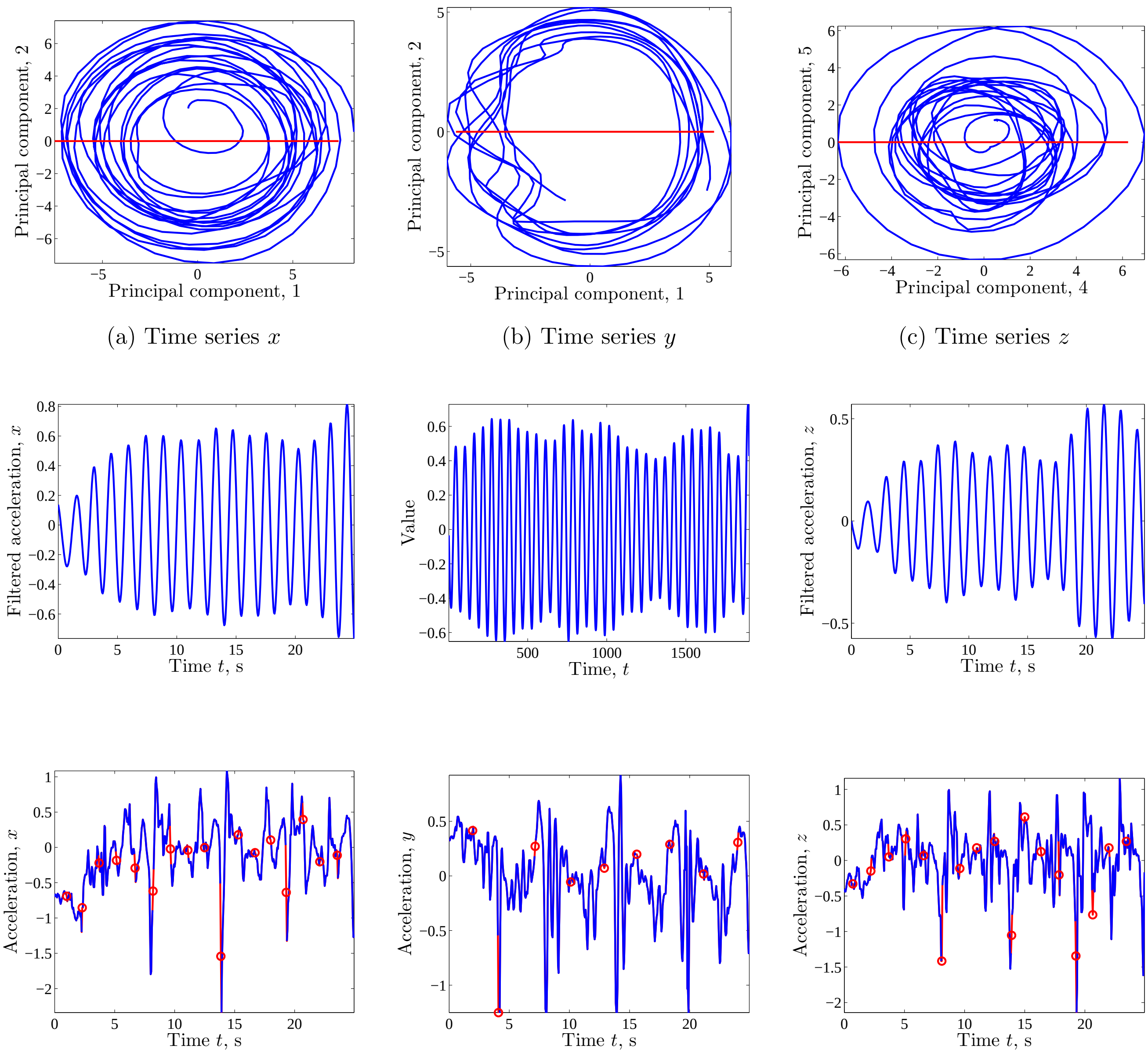

Fig. 7. Accelerometer-based time series of walking: Selected principal components, segmentation of the time series, obtained by cutting the principal components trajectory (red circles mark ending/starting points of each segment).

Automatic selection of the fundamental pair of principal components

The procedure of automatic selection utilized in the paper is based on the method proposed in the paper [20]. The idea behind this method is to detect principal components, periodical with the same frequency comparing their spectral densities.

For the time series X the Digital Fourier Transform is given by:

$$x(i) = \sum_{j=1}^{m} f(k) w_m^{-(k-1)(i-1)}, \quad w_m = \exp(-2\pi \mathrm{i}/m). \tag{6}$$

In this paper the spectral densities are computed with Fast Fourier Transform. The top row of the Figure 9 pictures the spectral density of the time series X: the dependencies of the amplitudes $|f(k)|$ on the frequencies $M(k-1)/m$, where

$$f(k) = \frac{1}{m} \sum_{i=1}^{m} X(i) w_m^{-(k-1)(i-1)},$$

and the value M equals to the number of samples per second.

As it was shown in the Section “Principal component selection” the periodical pair of principal components consists of two periodical functions differing only in their phase. The spectral densities of such principal components have their maximums of amplitude $|f(k)|$ in the same frequencies $M(k-1)/N$. The reasonings on the asymptotical equality of eigenvalues of such pairs allow to consider only consequential principal components.

Let k_j denote the number of frequency, corresponding to the maximum value $|f^j(k)|$ of principal component $\mathbf{y}_j$

$$k_j = \underset{k \in \{0,\dots,m-1\}}{\arg\max} |f^j(k)|.$$

Due to the noisiness or signal modulations, the spectral density may become “smeared”: the peaks become less sharp and precise and the arguments k_j and k_{j+1} may not exactly coincide even for matching principal component. We construct the set of candidate pairs $(\mathbf{y}_j, \mathbf{y}_{j+1})$, $j \in \mathcal{J}$, that have close arguments k_j

$$\mathcal{J} = \{j \in \{1, \dots, m-1\} \,|\, |k_j - k_{j+1}| < \varepsilon M\}.$$

The parameter ε controls accepted smearing of spectral density. The next step is to chose the pair corresponding to the fundamental periodic from this set. To chose the fundamental pair of principal components for human motion time series, we propose to use the knowledge about the frequency of the fundamental cycle of motion. As can be found in [?], for most types of motions this frequency composes about 0.5–2 Hz (about 1 Hz for walking). Thus, when choosing a pair of principal components we have to look for a pair with the values $M(k_j-1)/m$, $M(k_{j+1}-1)/m$ of peak frequencies close to 1 Hz. When no assumptions on the fundamental frequency can be made, the algorithm will chose a pair with minimum nonzero frequency.

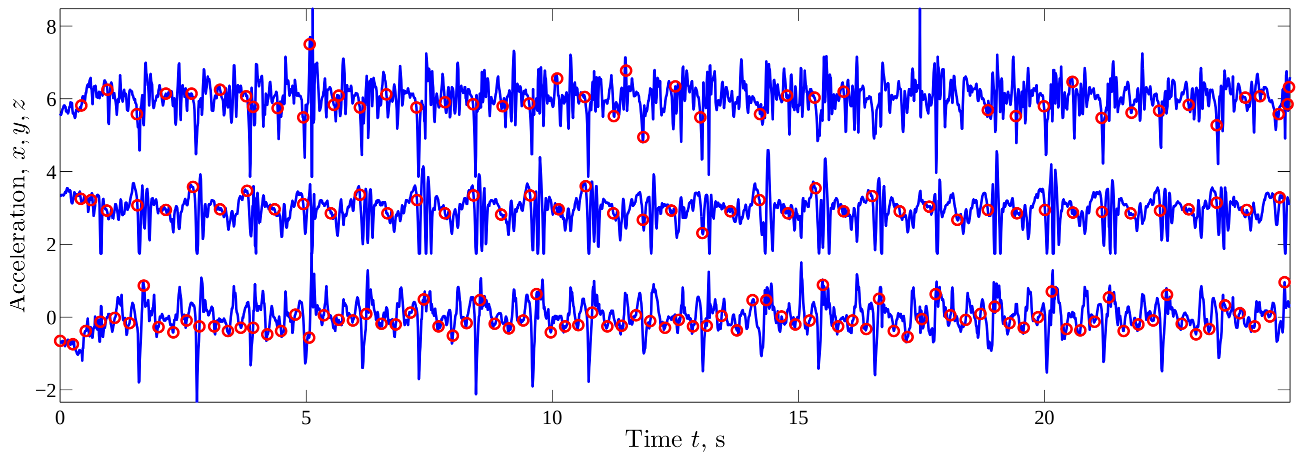

Fig. 8. The results of period extraction trough cutting the PC trajectory with its symmetry axis. The red circles mark the starting/ending points of the extracted periods.

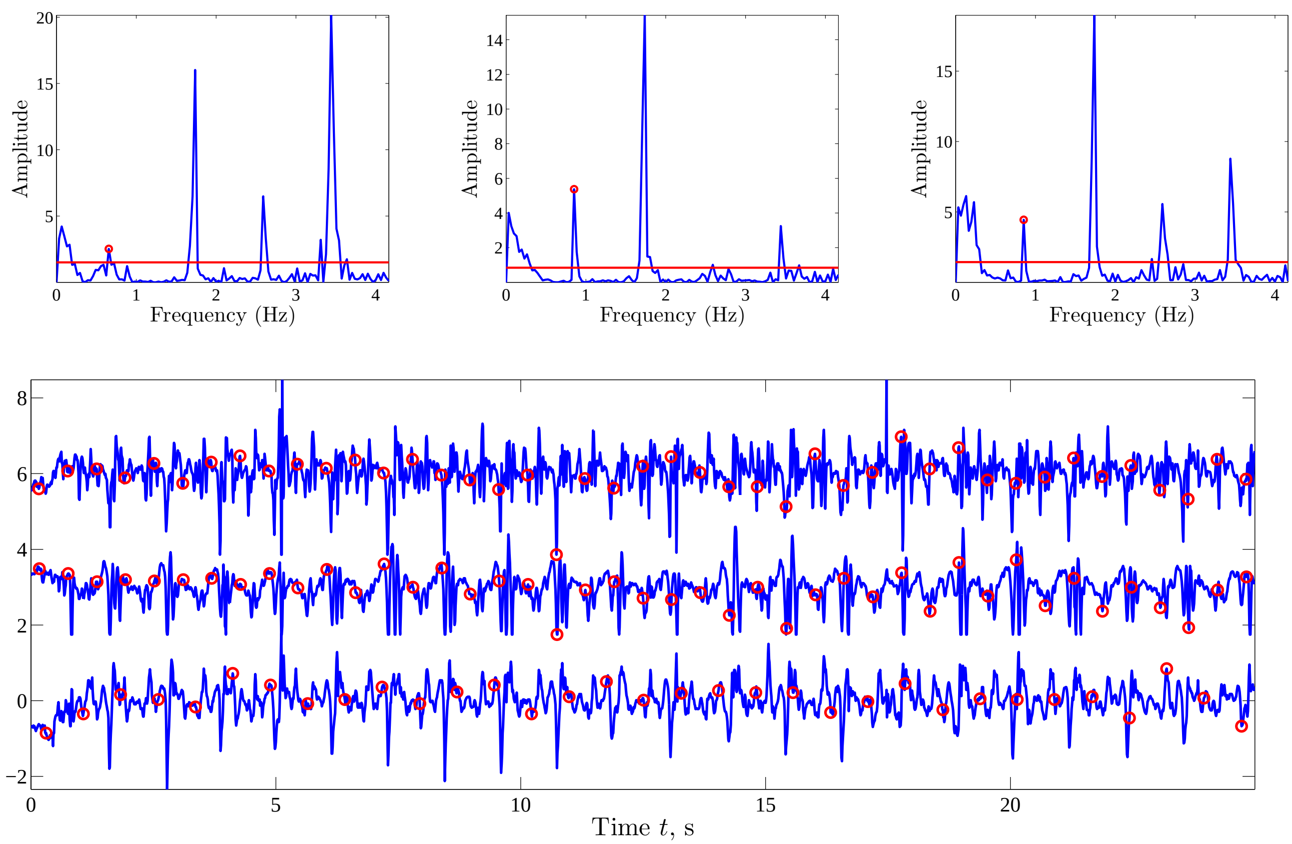

Fig. 9. Spectral density of the studied time series (acceleration along the dimensions x, y, z). The results of period extraction based on the filtered time series, reconstructed using the chosen peak frequency.

Example 3: period extraction on manually segmented accelerometer data. For qualitative evaluation of the results of periods extraction we provide the experiment on manually segmented data. The studied data contains acceleration measurements for different types of human motion. The time series are pictured in the first row of the Figure 6. The colors denote spatial dimensions: blue for the acceleration along the x axis, red and green — for y and z. As one can see, the time series for dimensions y and z don't allow to distinguish the steps of right and left limbs. Because of this, the periods extracted for y and z are two times shorter than the periods for x, since the longitudinal acceleration depends more on the side. The same observations hold for the time series for other types of activity. This allows us to suggest the following strategy of agreeing the results of the period extraction: let us consider the extracted period of the longitudinal acceleration the basis and use the partitions of time series for y and z to fine-tune the results. Of course, this might work only if the method of taking measurements (i.e, the placement of accelerometer, the dimensions represented by time series, etc.) is known.

Example 4: period extraction on the accelerometer data. The Figure 7 illustrates the results of implementing the same procedure to the accelerometer data (Fig. 1 — walking). The pictures in the top row of the Figure 7 shows the trajectories $(\mathbf{y}_1, \mathbf{y}_2)$ of the selected principal components for time series for each spatial dimension, divided by the estimated symmetry axis. The middle row demonstrates the time series, rebuild with the selected pair of principal components. The pictures in the bottom row show the results of segmentation. The picture on the Figure 8, below, gives the pieces of the segmented time series in a larger time scale (25 seconds). In this experiment, it is the time series for y acceleration that has a twice longer mean period. This happens because the different data sets have different ways of measuring acceleration (for example, the accelerometer can be placed on the wrist or on the waist). The Figure 9 demonstrates how the assumptions about fundamental frequency can be used to increase the quality of fundamental period extraction of human motion time series. For each time series the closest to 1 Hz peak frequency was chosen (the chosen frequencies are marked with red circles). The original time series can be filtered using (6) so that the filtered time series contains only peak frequencies that are close to the expected value. This filtered time series is then used to compute the principal components and define the ending points of the extracted periods (the Fig. 9, below). The segmentation built this way is more sparse: the average lengths of extracted periods, listed in the table 1, doubled after the filtration procedure. The table (1) lists mean lengths of the extracted periods for several data sets.

Table 1. The mean length of the extracted period.

Time series	x	y	z	PC pair
Walk 1	0.2775	0.3328	0.3725	(1 2), (1 2), (4 5)
Walk 1 (FFT)	0.7418	0.6390	0.6150	(1 2), (1 2), (1 2)
Jog 1	3.3026	1.6471	1.7176	(1 2), (1 2), (1 2)
Skip 1	2.5222	2.1428	2.2601	(1 2), (1 2), (1 2)
Walk 3	1.3803	1.5270	1.5679	(3 4), (2 3), (4 5)
Slow walking	2.6000	1.5400	1.4222	(1 2), (1 2), (1 2)
Skipping	1.1214	0.7573	0.6749	(1 2), (1 2), (1 2)
Jogging	1.2375	0.7225	0.6871	(1 2), (1 2), (1 2)

Alternative methods of estimating the period length

In this section we describe some classical methods of estimating the period length and provide a comparison of the results we obtained applying the proposed method and the classical alternatives to the walking data set.

The Least Squares Estimation. To obtain the least squares estimation one fits the model

$$X(i) = \sum_{q=1}^{Q} A_q \cos(w_q i) + B_q \sin(w_q i)$$

best according to the residual sum of squares

$$\{\mathbf{w},\, \mathbf{A},\, \mathbf{B}\} = \operatorname*{arg\,min}_{\mathbf{w}, \mathbf{A}, \mathbf{B}} \sum_{i=1}^{m} \left(\tilde{X}(i) - \sum_{q=1}^{Q} A_q \cos(w_q i) + B_q \sin(w_q i)) \right)^2.$$

Let us consider for simplicity the case of $q = 1$, then, as shown in [1] the residual squares functional can be modified to an asymptotically equal one (for $0 < w < \pi$)

$$\{\mathbf{w},\, \mathbf{A},\, \mathbf{B}\} = \operatorname*{arg\,min}_{\mathbf{w}, \mathbf{A}, \mathbf{B}} \sum_{i=1}^{m} X^2(i) - 2\sum_{i=1}^{m} A\cos(wi) + B\sin(wi) + \frac{2}{m}(A^2 + B^2) \qquad (7)$$

to provide the following estimations $\hat{A}$ and $\hat{B}$ of A and B

$$\hat{A}(w) = \frac{2}{m}\sum_{i=1}^{m} X\cos(wi), \quad \hat{B}(w) = \frac{2}{m}\sum_{i=1}^{m} X\sin(wi).$$

Maximization of the periodogram. Note that the functional (7) can be written as following

$$\sum_{i=1}^{m} X^2(i) - 2\sum_{i=1}^{m} A\cos(wi) + B\sin(wi) + \frac{2}{m}(A^2+B^2) = \sum_{i=1}^{m} X^2(i) - \frac{2}{m}\left|X(i)e^{-\mathbf{i}wi}\right|,$$

which means the the minimization (7) is equivalent to maximizing the periodogram [2] $\frac{2}{m}\left|X(i)e^{-\mathbf{i}wi}\right|$. In this paper we do not distinguish between these estimators, considering jnly the given by (7) and treating it as least squares estimation.

The cross correlation estimation. To obtain the cross correlation estimation, we partitioned the time series into $T(|S|)$ segments S_t of same length $|S|$ in the predefined range $[|S|_{\min}, |S|_{\max}]$ and computed the average Pearson's correlation coefficient

$$\mathrm{Corr}(X, |S|) = frac1T - 1\sum_{t=1} T - 1\rho(S_t, S_{t+1})$$

between the neighboring segments S_t, S_{t+1}. The cross correlation estimation of the period is then given by

$$\frac{1}{w} = \arg\max_{|S|} \mathrm{Corr}(X, |S|).$$

We chose the $|S_{\max}| = m/4$, meaning that the time series should contain at least four periods, and the minimum number $S_{\min}$ of points in a period to three.

Comparing the results. The results of period length estimation with the methods, presented above as well as the proposed method are displayed on the histogram on the picture 10. The alternative methods are labeled “LSE” and “Correlation”. The “PCs + FFT” stands for the estimation obtained by maximizing periodogram of a chosen principal component approximated through FFT. The “Segmentation” estimation equals the average length of the extracted fundamental periods. On the histogram, the distribution of the errors of each method are plotted. For each time series in the set the total number T of steps (periods) was known, thus the error is calculated as the absolute difference

$$m\left|\frac{1}{\hat{T}} - \frac{1}{T}\right|.$$

Though the proposed method (“Segmentation”), along with the “PCs + FFT” provide better results, the overall performance of these estimators is unsatisfying due to significant aperiodicity of the studied data.

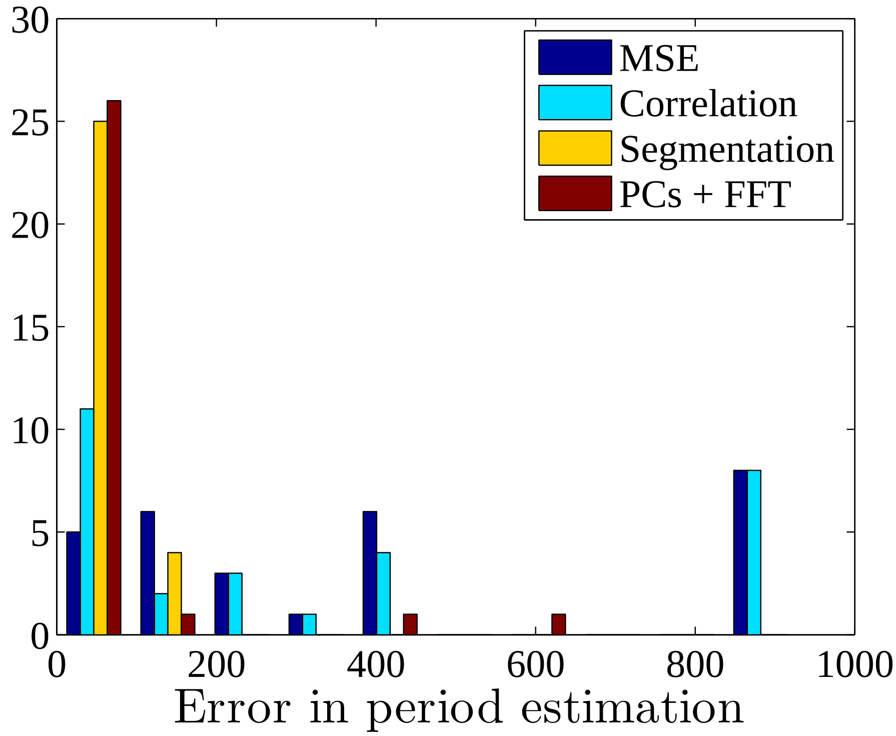

Fig. 10. Comparison of the results of period length estimation.

Conclusion

The paper discusses the problem of partitioning human motion time series into interpretable segments. Solving this problem is an important step to analyzing sensor-based time series of human motion. To provide the interpretability of the segmentation, the authors propose to extract segments, considering the nature of the studied data and define a segment as a period of fundamental cycle of motion. The ending points of the periods are defined through cutting the trajectory of a pair of principal components of the trajectory matrix. The authors describe a procedure of selecting a pair of principal components, corresponding to the fundamental periodic component of the time series.

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